

Nonlinear processes in rotating fluids: a report on Euromech 56

By D. J. ACHESON,

Mathematical Institute, University of Oxford

D. G. ANDREWS,

Meteorological Office, Bracknell, Berkshire†

I. C. WALTON

Mathematics Department, Imperial College, London

AND L. M. HOCKING

Mathematics Department, University College, London

(Received 7 June 1975)

A European Mechanics Colloquium (Euromech 56) on ‘Nonlinear processes in rotating fluids’ was held at University College, London from 14–18 April 1975. It was attended by 77 mathematicians, engineers and physicists from 14 countries, and 33 papers were presented and discussed. The topics included motions in homogeneous and stratified rotating fluids, with applications to the atmosphere, the oceans, the earth’s core and centrifuges. Brief summaries of the papers presented are contained in this report.

1. Introduction

The primary aim of Euromech 56 was to discuss recent progress towards the understanding of fluid motions in a rotating frame when the Rossby number is not negligibly small. When inertial (as well as viscous and Coriolis) forces are taken into account, asymptotic analysis of the equations of motion for a homogeneous fluid reveals a variety of boundary layers and detached shear layers. The important role played by these layers in steady-state solutions, and in the manner in which they are achieved from rest or from another steady state (e.g. the ‘spin-up’ problem), was discussed during the Colloquium and the relevant papers are described briefly in §2.

The major part of the Colloquium was, however, taken up by the consideration of inhomogeneous fluids. The combined effects of rotation and gravity acting on a stratified fluid are notoriously complex. A central topic in their study is the theory of baroclinic waves, and such waves were discussed (§3) in relation to both laboratory experiments and the atmosphere (or numerical models of it). Other papers dealing with specifically meteorological problems are described in §4. Two important features of nonlinear waves are the mean flows associated with them and the corresponding transfer properties of the

† Present address: Department of Geophysics, University of Reading.

waves. Results for a rotating stratified medium are described in §5. If the study of rotating fluids is to be extended to allow discussion of motions in the earth's core, and thus of the maintenance of the geomagnetic field against ohmic decay, the further complication of hydromagnetic effects must be included (§6).

The motivation for the understanding of rotating fluids stems not only from geophysical applications, but also from industrial and medical applications, notably the use of centrifugal separation devices (§7). In gas centrifuges used for isotope separation the rotation rate may be so high that compressibility becomes a crucial feature of the motion.

2. Homogeneous fluids

Steady motion

Detached shear layers parallel to the rotation axis occur frequently in rotating fluids. Linear viscous shear layers have been described by Stewartson (1957) for a simple split-disk configuration. The fluid is contained between two infinite horizontal disks rotating about the same vertical axis with the same angular velocity Ω , but an inner part of each disk is allowed to rotate independently with angular velocity $\Omega(1 + \epsilon)$, say. M. Israeli (Technion, Haifa) presented a numerical study of the nonlinear equations for this motion and demonstrated that, for an Ekman number $E \approx 10^{-3}$, viscous shear layers could still be discerned even when the Rossby number $|\epsilon|$ was as large as unity. The flow was not radically different from the linear one. He also gave an improved linear theory based on a local Ekman number. S. H. Smith (University of Toronto) discussed the appropriate shear-layer thickness when $|\epsilon| \gg E^{\frac{1}{2}}$. With $\epsilon > 0$, a balance between inertial and viscous terms indicates a layer of thickness $E^{\frac{2}{3}}|\epsilon|^{-\frac{1}{3}}$, in agreement with Bennetts & Hocking (1973). When $\epsilon < 0$, a layer of thickness $|\epsilon|$ is appropriate, again as found by Bennetts & Hocking, but there is no possible inner layer, analogous to the linear $E^{\frac{1}{2}}$ layer, so that some breakdown of the flow must occur before $|\epsilon|$ becomes large compared with $E^{\frac{1}{2}}$.

Developments in the theory of Couette-flow instability were given in two talks. Th. Alziary & M. G. Grillaud (University of Poitiers) presented numerical solutions for the flow between two vertical coaxial rotating cylinders, taking into account the presence of the end walls. They showed that the number of vortices increases with the Reynolds number R for $10 < R < 4 \times 10^3$ and that there may be as many as 10 when the aspect ratio is 10 and $R = 2500$. The results are in good agreement with experiments and with the theory for infinite cylinders at the same Reynolds number. M. Wimmer (University of Karlsruhe) presented the results of his experimental studies of the instability of the flow in a spherical annulus. With the outer sphere at rest and a small gap between the spheres, the number of vortices present increases with R , but for a wide gap vortices only occur near the equator and their number does not increase with R . Five different modes are possible, consisting of one or two vortices on either side of the equator (stationary or oscillating) and a spiral vortex near the pole.

Unsteady motion

A paper describing an experimental investigation of an unsteady Taylor column was presented by H. G. Moll (University of Berlin). When a small sphere moves freely under gravity in a rotating fluid, the inclination of the Taylor column may be traced from the distortion of a dyed fluid filament. Even when the Rossby number is as high as 2 the results are in good agreement with those of Hide, Ibbetson & Lighthill (1968) for steady motion at low Rossby numbers. J. A. King-Hele (University of Manchester) described his work with R. W. Paulson on the formation of boundary layers, of the kind found on west coasts in mid-latitudes, by the incidence of Rossby waves. While Lighthill (1969) has considered the limitation of the boundary-layer intensification by viscous effects, the limitation is achieved by nonlinear effects in the parameter range considered here. In the first of two model problems, King-Hele described how the nonlinear response to a suddenly imposed constant wind stress could be found analytically, but the approach to a steady state could be found only near a stagnation point. In his second problem, the response of an inertial boundary layer to an incoming Rossby wave could be described, as well as the final steady state. Rossby waves of periods in excess of one month were shown to be trapped by the steady inertial current.

The effects of a sudden increase in the angular velocity of a fluid container were described in two papers. K. D. Aldridge (University of Alberta, Edmonton) reported his experimental investigations of the inertial oscillations set up when a cylindrical container is given a sudden periodic perturbation. He found only a weak dependence of the eigenfrequencies on the aspect ratio. D. R. Moore (University of Cambridge) & A. D. Weir (Imperial College, London) presented numerical solutions for the axisymmetric spin-up of a spherical container, the nonlinear results being qualitatively similar to the linear ones. A problem raised by recent numerical studies of planetary atmospheres is whether the numerical modelling of the boundaries leads to a spurious flux of angular momentum, but in the results reported it appeared to be small.

3. Baroclinic waves

Baroclinic waves, which are important in the understanding of many atmospheric processes, have been extensively studied through experiments in a differentially heated rotating fluid annulus. P. J. Mason (Meteorological Office, Bracknell) showed films demonstrating the various types of flow which are observed to occur (axisymmetric flow, steady waves, vacillating waves, irregular flow), described in detail in Hide & Mason (1975). He discussed the effects of sloping upper and lower boundaries and showed that, if they remain parallel, the slope of the particle trajectories, and hence the energy-releasing mechanism, is directly affected. If they slope in opposite senses, however, processes similar to a beta-effect are introduced. H. Leach (Meteorological Office, Bracknell) described annulus experiments with bottom topography. In the absence of free baroclinic waves, the disturbance produced by an obstacle is steady and

confined to the lower part of the fluid; a closed circulation appears near the obstacle. If free baroclinic waves are present, the topographic disturbances extend throughout the depth of the fluid and exchange energy with the dominant drifting free wave. The transitions between the flow regimes were also shown to be affected by the topography. R. Hide, P. J. Mason & R. A. Plumb (Meteorological Office, Bracknell) discussed the azimuthal spectra obtained in regular and irregular flows in an annulus. They showed that the nominally steady wave flow is dominated by a component of wavenumber m (of nearly constant amplitude) and its harmonics, together with rather weaker side bands of wavenumbers $m \pm 1$, having time-varying amplitude. The behaviour of the frequency and relative phases of these waves is indicative of nonlinear side-band resonance. On the other hand, vacillating flows exhibit a dominant component and its harmonics, all of which display a substantial harmonic time variation. The behaviour of spectral components at the transition to irregular flow was shown to be consistent with the hypothesis that this transition is due to an instability similar to the barotropic instability of Rossby waves.

R. K. Smith (Monash University, Melbourne) described a theory of vacillating finite-amplitude baroclinic waves in a two-layer quasi-geostrophic zonal flow. He extended the work of Pedlosky (1971, 1972) to include an appropriate side-wall boundary condition on the zonal mean flow, and demonstrated the existence of stable periodic solutions for vanishingly small viscosity. He also discussed the case where viscous effects are present but are small on the time scale of the growth of an incipient wave, and obtained conditions under which a possible steady solution is locally stable or unstable. Numerical integrations of the amplitude evolution equations in this viscous regime have failed to yield periodic solutions. Numerical simulations of amplitude and shape vacillation were outlined by A. Quinet (Royal Meteorological Institute of Belgium, Brussels), who applied spectral methods to the two-layer quasi-geostrophic model. In amplitude vacillation, there is an alternation between the effects of a direct Hadley cell and quasi-horizontal geostrophic eddies; contrasts with linear theory were noted. The importance of barotropic processes in shape-vacillation (or tilted-trough vacillation) was discussed, together with the associated energetics and momentum transfer.

The instability of a baroclinic zonal flow is usually studied on the basis of the quasi-geostrophic equations, which are valid only if the Rossby number is small. This condition is not satisfied in many atmospheric flows, in particular, in nonlinear baroclinic waves. B. J. Hoskins (University of Reading) described an approximation based on the smallness of the fluid *acceleration*. This gives a set of equations which, by means of a co-ordinate transformation, may be studied analytically and numerically (Hoskins 1975). These approximate equations are capable of describing many of the nonlinear aspects of baroclinic waves, such as the formation of fronts. A. J. Simmons (University of Reading) described his work with A. Hollingsworth on a spectral numerical study of nonlinear baroclinic instability on a sphere. The finite-amplitude solutions exhibit marked distortions from those predicted by linear theory, with fronts, occlusions and

intense low pressure centres being formed. However, the overall patterns of heat and momentum transfer remain similar to those deduced from linear theory.

J. S. A. Green (Imperial College, London) discussed how some of the gross features of atmospheric motions may be accounted for in terms of the transfer properties of finite-amplitude baroclinic waves. The uncertainties involved in the mixing-length hypothesis are avoided by reference to a finite rearrangement of the atmosphere, in which entropy and potential vorticity are conserved. In this way a realistic distribution of surface westerlies and easterlies may be produced.

4. Other meteorological problems

During the meeting, a number of papers on meteorological topics were presented, in addition to those on baroclinic waves described in the previous section. J. Egger (University of Munich) described the use of a five-layer numerical weather prediction model to study the mechanism by which cyclones are generated in the lee of north-south mountain barriers as 'parent lows' approach from the windward side, move north and decay. He concluded that the influence of the warm air ahead of the parent low is the primary cause of the initial fall in pressure, with vorticity advection aloft being important only during the final stages of cyclogenesis. M. Dunst (University of Hamburg) described numerical integrations of the shallow-water equations on a sphere, and examined the breakdown of a basic super-rotation, with initial conditions provided by a Rossby-Haurwitz wave. For relatively small initial zonal kinetic energy, a poleward angular-momentum transport occurred, with jet-stream-like currents forming by day 4 at 32° N. For larger initial zonal kinetic energy, angular momentum was fed southwards, leading to a jet at 9° N by day 6. Dunst also described laboratory experiments in which the behaviour of quasi-two-dimensional vortices was studied in a rotating cylinder with a simulated beta-effect.

D. Etling (Technical University, Darmstadt) outlined a linear stability analysis of a steady-state turbulent atmospheric boundary layer with eddy viscosity depending on the thermal stratification (Wippermann 1972). The resulting eigenvalue problem was solved numerically, and perturbations took the form of horizontal rolls inclined to the geostrophic flow at angles depending on a surface Rossby number. According to this theory, with the relevant parameters given reasonable values, the atmospheric boundary layer is unstable. P. Kočíková (Czechoslovak Academy of Sciences, Prague) described the secondary vorticity centres which may appear behind cold fronts in extra-tropical cyclones. Such centres are difficult to detect observationally, but their neglect may lead to significant errors in mesoscale weather prediction. She showed how the method of writing the vorticity equation in intrinsic co-ordinates, following Lakshminarayana & Horlock (1973), can be used in a numerical model to study the effects of such phenomena.

R. Kh. Zeytounian (University of Lille) considered in some detail the approxi-

mations involved in the use of the quasi-geostrophic equations to describe the unsteady flow of a rotating, viscous, compressible and thermally conducting fluid at low Rossby number.

5. Waves and mean flows

D. G. Andrews (Meteorological Office, Bracknell) described his work on mean flows forced by upward-propagating equatorial Kelvin and Rossby-gravity waves in a basic zonal shear flow. A consistent multiple-scale analysis to second order in the wave amplitude confirms that the latitudinal average of the forced mean zonal acceleration equals that predicted by the semi-heuristic arguments of Holton & Lindzen (1972) in their theory of the quasi-biennial oscillation, but reveals, in addition, that the latitudinal *structure* of the easterly acceleration forced by Rossby-gravity waves depends critically on the dissipation mechanisms involved. The structure has either a single or a double peak, according as Rayleigh friction or Newtonian cooling is the more important wave-damping mechanism.

R. Grimshaw (University of Melbourne) discussed the propagation of a non-linear internal gravity wave packet in an unbounded rotating fluid. The evolution of the wave packet is described by conservation of wave action (Bretherton & Garrett 1968; Bretherton 1971), and the conceptual value of formulating the development of the mean flow field in Lagrangian rather than Eulerian terms was emphasized, for the effect of the waves may then be ascribed simply to the divergence of an appropriate radiation-stress tensor in the mean momentum equation. Also, a Kelvin circulation theorem was derived for the Lagrangian mean velocity. M. E. McIntyre (University of Cambridge) described a similar theory and considered, in particular, the mean flow forced by upward-propagating inertio-gravity waves in a zonal channel on an f -plane. He indicated ways in which the use of a general averaging operator allows a corresponding formulation of the Lagrangian mean flow, and also pointed out that errors resulting from application of the 'photon' concept to waves in fluids (McIntyre 1973) are due to a confusion between physical momentum and a quantity analogous to the 'pseudo-momentum' used by solid-state physicists.

6. Geomagnetic dynamo theory

In the dynamo theory of the maintenance of the earth's magnetic field against ohmic decay, the energy source of the motion has long been a subject of controversy, thermal convection and precession hitherto appearing to be the only serious candidates. P. H. Roberts (University of Newcastle upon Tyne) discussed his work with K. Stewartson on thermally convective motions in a horizontal layer of fluid rotating about a vertical axis in the presence of a co-rotating horizontal magnetic field. In the weakly nonlinear regime, they find that the two roll systems which are maximally unstable on linear theory interact to create a geostrophic motion parallel to the applied magnetic field. An interesting feature that arises from their analysis of the stability of one roll system

with respect to the other (presumed to have a very much smaller initial amplitude) is the hint of a self-adjustment mechanism maintaining Lorentz and Coriolis forces of comparable magnitude (as they are commonly believed to be in the earth's core), for it is then that both roll systems must co-exist. This is thought to provide a situation favourable to the regeneration of the magnetic field.

M. R. E. Proctor (University of Cambridge) described his work with W. V. R. Malkus (1975) on ' α -effect' dynamos, in which large-scale magnetic fields are maintained by small-scale motions possessing mean 'helicity' (Moffatt 1972). In previous work, the growth of such fields is limited by reduction in the helicity of these small-scale motions as the field increases, but Proctor pointed out that large-scale magnetic fields with global boundary conditions cannot be force-free and will thus drive large-scale motions as they grow. Equilibration of the process can then arise from an increased dissipation rate, primarily ohmic, due to distortions of the magnetic field caused by these large-scale motions. J. A. Jacobs (University of Cambridge) presented his recent work with Rochester *et al.* on the question of whether precession, rather than thermal convection, drives the dynamo. The precessional power input to the core can be estimated in terms of the dissipative part of the core-mantle coupling and the tilt-over angle (the inclination between the angular momentum vectors of the core and the mantle). Jacobs pointed out that allowance must be made for the dependence of the coupling strength on the diurnal frequency of the precession-induced core flow relative to the mantle, and argued that the papers of Malkus (1963) and Stacey (1973) contain serious flaws, particularly in their estimates of the tilt-over angle. His conclusions indicate that precession fails by at least two orders of magnitude to provide the power necessary to drive the geodynamo.

7. Fluid dynamics of centrifugal separation devices

Centrifugal forces are exploited in machines developed to separate materials. D. B. Ingham (University of Leeds) described his work with M. I. G. Bloor on the fluid mechanics of industrial cyclones, which are used to separate small solid particles from a liquid of lower density. The suspension is injected tangentially at high speed into a conically shaped vessel (apex lowermost) at its largest diameter. This drives a rapidly rotating inviscid 'core' flow in which most of the injected fluid initially descends, rises near the cone axis and thence leaves the cyclone through an outlet at the top. The particles, however, drift outwards under the centrifugal force, descend through the boundary layer on the side wall of the cone and then leave via a small orifice at its vertex. Extending their previous work by analysing this side-wall boundary layer and relaxing a simplifying assumption made by Wilks (1968) in its treatment, they found motions towards the vertex in the boundary layer four or five times faster than the meridional motions in the core. In reply to questions from the audience, they emphasized that, in contrast to the swirl-atomizer problem, these meridional core motions must be allowed for when analysing the side-wall boundary layer.

G. Langbein (URENCO, Marlow) discussed the use of gas centrifuges for separating uranium isotopes. These devices are rotated as fast as the strength of the walls will allow, with peripheral velocities of the order of several hundreds of metres per second. The fluid-dynamical problems that were described included that of generating a countercurrent by thermal or mechanical means to aid in the separation process. Langbein also gave a brief report on the work currently being carried out in this field by the joint Anglo-Dutch-German group, which was set up in 1971.

Compressibility of the gas in such rapidly rotating systems has important effects. F. H. Bark (Royal Institute of Technology, Stockholm) described his work with T. H. Bark on compressible $E^{\frac{1}{2}}$ and $E^{\frac{3}{2}}$ boundary layers in a rotating isothermal gas. He presented numerical solutions for the velocity field and, for the $E^{\frac{1}{2}}$ layer, an exact boundary-layer solution. When the density scale height was much greater than the appropriate boundary-layer thickness, uniformly valid asymptotic solutions were obtained; these agreed well with the other solutions, and contrasted with the results of Nakayama & Usui (1974), which are not uniformly valid. S. B. Berndt (Royal Institute of Technology, Stockholm) addressed the problem of whether purely axial relative flow is possible for a compressible fluid in a rapidly spinning tube. He performed an expansion in the small Mach number m based on the axial velocity, postulated the flow to vary slowly in the axial direction and obtained an azimuthal velocity of order mM , where M is the Mach number based on the peripheral velocity. Thus the flow cannot be exactly axial, but in the parameter range relevant to the gas centrifuge, the length scale of axial variations is enormous (several kilometres!), and the departure from axial flow is negligible.

Separation of biological samples can be achieved by means of a 'coil planet centrifuge', originally proposed by Ito & Bowman (1971). I. A. Sutherland (National Institute for Medical Research, Mill Hill) described this apparatus, in which a tube wound in the form of a long helical coil rotates about the (vertical) axis of the helix, while this axis of rotation itself rotates with an equal and opposite angular velocity about a distant fixed vertical axis. If the coil is filled with the heavier of two immiscible solvents and the lighter solvent is pumped through, a biological sample can then be injected and its constituents separated as a result of the solvent phase distribution set up in the coil. Sutherland outlined the advantages of the system over previous countercurrent chromatography techniques and reported its success in the separation of amino acids and polyene antibiotics. B. J. Hammond (National Institute for Medical Research, Mill Hill) described the results of a numerical simulation of the motion of a spherical particle in a simplified model of this system, assuming inelastic collisions with the boundaries and Stokes's formula for the drag. The possible particle trajectories range from small-scale circular motion to 'orbital' motion round the axis of the helix and even to a 'trapped' mode in which, having once struck the boundary, the particle never leaves it. Participants were treated *in situ* to a look at the centrifuge in action.

REFERENCES

(An asterisk by a name indicates a lecture given at the Colloquium)

- ALDRIDGE, K. D.* An experimental study on inertial oscillations of a rotating cylinder of fluid during spin-up from rest.
- ALZIARY, TH. & GRILLAUD, M. G.* Computation of flows in a rectangular annular cavity.
- ANDREWS, D. G.* Mean flows forced by waves in the equatorial stratosphere.
- BARK, F. H. & BARK, T. H.* On vertical boundary layers in a rapidly rotating gas.
- BENNETTS, D. A. & HOCKING, L. M. 1973 On nonlinear Ekman and Stewartson layers in a rotating fluid. *Proc. Roy. Soc. A* **333**, 469–489.
- BERNDT, S. B.* Almost axial gas flow in a spinning annular tube.
- BLOOR, M. I. G. & INGHAM, D. B.* Some aspects of the fluid mechanics of the industrial cyclone. (See also *Trans. Inst. Chem. Engng*, **51** (1973), 36–41; **53** (1975), 1–16.)
- BRETHERTON, F. P. 1971 The general linearised theory of wave propagation. *Lect. Appl. Math.* **13**, 61–102.
- BRETHERTON, F. P. & GARRETT, C. J. R. 1968 Wavetrains in inhomogeneous moving media. *Proc. Roy. Soc. A* **302**, 529–544.
- DUNST, M.* Investigations on the connexion between two-dimensional momentum transfer and jet-stream formation.
- EGGER, J.* Numerical experiments on lee cyclogenesis.
- ETLING, D.* On the stability of the atmospheric boundary-layer flow.
- GREEN, J. S. A.* Transfer properties of baroclinic waves.
- GRIMSHAW, R.* Nonlinear internal gravity waves in a rotating fluid. (See also *J. Fluid Mech.* **71** (1975).)
- HAMMOND, B. J.* The motion of a particle in a planetary centrifuge.
- HIDE, R., IBBETSON, A. & LIGHTHILL, M. J. 1968 On slow transverse flow past obstacles in a rapidly rotating fluid. *J. Fluid Mech.* **32**, 251–272.
- HIDE, R. & MASON, P. J. 1975 Sloping convection in a rotating fluid. *Adv. in Phys.* **24**, 47–100.
- HIDE, R., MASON, P. J. & PLUMB, R. A.* High Taylor number thermal convection in a rotating fluid annulus subject to a horizontal temperature gradient: spectral characteristics of the non-axisymmetric flow regimes.
- HOLLINGSWORTH, A. & SIMMONS, A. J.* Some nonlinear properties of baroclinic waves in a numerical model.
- HOLTON, J. R. & LINDZEN, R. S. 1972 An updated theory for the quasi-biennial cycle of the tropical stratosphere. *J. Atmos. Sci.* **29**, 1076–1080.
- HOSKINS, B. J. 1975 The geostrophic momentum approximation and the semigeostrophic equations. *J. Atmos. Sci.* **32**, 233–242.
- HOSKINS, B. J.* Nonlinear baroclinic waves and the formation of fronts.
- ISRAELI, M.* On a nonlinear effect in vertical shear layers in a rotating fluid.
- ITO, Y. & BOWMAN, R. L. 1971 Countercurrent chromatography with flow-through coil planet centrifuge. *Science*, **173**, 420–422.
- KING-HELE, J. A. & PAULSON, R. W.* Unsteady barotropic inertial boundary layers.
- KOČÍKOVÁ, P.* Secondary vorticity centres as centres of secondary vorticity (or stream-wise vorticity).
- LAKSHMINARAYANA, B. & HORLOCK, J. H. 1973 Generalized expressions for secondary vorticity using intrinsic co-ordinates. *J. Fluid Mech.* **59**, 97–115.
- LANGBEIN, G.* Survey about the state of the art in fluid dynamics in the Tripartite Project Isotope Separation with gas centrifuges.
- LEACH, H.* Thermal convection in a rotating fluid annulus: some effects due to bottom topography.

- LIGHTHILL, M. J. 1969 Dynamic response of the Indian Ocean to onset of the Southwest Monsoon. *Phil. Trans. A* **265**, 45–92.
- MCINTYRE, M. E. 1973 Mean motions and impulse of a guided internal gravity wave packet. *J. Fluid Mech.* **60**, 801–811.
- MCINTYRE, M. E.* Radiation stress and wave pseudo-momentum.
- MALKUS, W. V. R. 1963 Precessional torques as the cause of geomagnetism. *J. Geophys. Res.* **68**, 2871–2886.
- MALKUS, W. V. R. & PROCTOR, M. R. E. 1975 Macrodynamics of α -dynamoes. *J. Fluid Mech.* **67**, 417–443.
- MASON, P. J.* Sloping convection in a container with sloping end-walls. (See also *Phil. Trans. A* **278** (1975), 397–445.)
- MOFFATT, H. K. 1972 An approach to a dynamic theory of dynamo action in a rotating conducting fluid. *J. Fluid Mech.* **53**, 385–399.
- MOLL, H. G.* The flow around a freely moving sphere in rotating water.
- MOORE, D. R. & WEIR, A. D.* Numerical simulations of axisymmetric spin-up in a sphere.
- NAKAYAMA, W. & USUI, S. 1974 Flow in rotating cylinder of a gas centrifuge. *J. Nucl. Sci. Tech. (Japan)*, **11**, 242–262.
- PEDLOSKY, J. 1971 Finite-amplitude baroclinic waves with small dissipation. *J. Atmos. Sci.* **28**, 587–597.
- PEDLOSKY, J. 1972 Limit cycles and unstable baroclinic waves. *J. Atmos. Sci.* **29**, 53–63.
- PROCTOR, M. R. E.* The macrodynamics of α -effect dynamoes in rotating fluids.
- QUINET, A.* Some nonlinear aspects of vacillation from numerical experiments.
- ROBERTS, P. H. & STEWARTSON, K.* Convective motions in a rotating magnetic layer. (See also *Phil. Trans. A* **227** (1974), 287–315; *J. Fluid Mech.* **68** (1975), 447–466.)
- ROCHESTER, M. G., JACOBS, J. A., SMYLLIE, D. E. & CHONG, K. F.* Can precession power the geomagnetic dynamo? (Submitted to *Geophys. J. Roy. Astr. Soc.*)
- SMITH, R. K.* On a theory of amplitude vacillation in baroclinic waves. (See also *J. Atmos. Sci.* **31** (1974), 2008–2011.)
- SMITH, S. H.* Nonlinear ‘split-disk’ problem.
- STACEY, F. D. 1973 The coupling of the core to the precession of the earth. *Geophys. J. Roy. Astr. Soc.* **33**, 47–55.
- STEWARTSON, K. 1957 On almost rigid rotations. *J. Fluid Mech.* **3**, 17–26.
- SUTHERLAND, I. A.* Countercurrent chromatography.
- WILKS, G. 1968 Swirling flow through a convergent funnel. *J. Fluid Mech.* **34**, 575–593.
- WIMMER, M.* Viscous fluid flow between concentric rotating spheres.
- WIPPERMAN, F. 1972 Universal profiles in the barotropic planetary boundary layer. *Beitr. Phys. Atmos.* **45**, 148–163.
- ZEYTOUNIAN, R. KH.* The unsteady flow of a rotating, heavy, viscous, compressible and thermally conducting fluid at low Rossby number (nonlinear asymptotic theory).